

Line Drawing Interpretation in a Multi-View Context (Supplementary Material)

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Details on the computation of $\mathcal{M}(l)$

We detail here the resolution of the quadratic minimization problem under linear constraints formulated in Section 4.3. We want to minimize the function ϵ

$$\epsilon = \sum_{i \in \mathcal{V}} \|RP_i - SP_i p_i\|^2 \quad (1)$$

under the following linear constraints

$$(\forall i \in \mathcal{F}) \sum_{j \in \mathcal{E}} c_{ij} \lambda_j v_j = 0 \quad (2)$$

Each vertex P_i is defined in homogeneous coordinates using

$$P_i = P_0 + \sum_{j \in \mathcal{E}} \delta_{ij} \lambda_j v_j \quad (3)$$

where P_0 is the reference vertex expressed in the camera frame as

$$P_0 = \begin{pmatrix} \alpha \hat{n} + \beta \hat{u} + \gamma \hat{v} \\ 0 \\ 1 \end{pmatrix} \quad (4)$$

where \hat{n} is the camera direction. P_0 is rewritten in a more compact form as

$$P_0 = \omega + \sum_{k=1}^2 \eta_k \hat{n}_k \quad \text{with} \quad \begin{cases} \eta_1 \hat{n}_1 = \begin{pmatrix} \beta \hat{u} \\ 0 \end{pmatrix} \\ \eta_2 \hat{n}_2 = \begin{pmatrix} \gamma \hat{v} \\ 0 \end{pmatrix} \\ \omega = \begin{pmatrix} \alpha \hat{n} \\ 1 \end{pmatrix} \end{cases} \quad (5)$$

By first replacing the expression of P_0 from Eq. 5 into Eq. 3, and then replacing the expression of P_i into Eq. 1, we can formulate ϵ as

$$\epsilon = \sum_{i \in \mathcal{V}} \|R\omega - S\omega p_i + \sum_{k=1}^2 \eta_k (R\hat{n}_k - S\hat{n}_k p_i) + \sum_{j \in \mathcal{E}} \delta_{ij} \lambda_j (Rv_j - Sv_j p_i)\|^2 \quad (6)$$

This can also be formulated as

$$\epsilon = \lambda^T A_0 \lambda + \lambda^T B_2 \eta + \eta^T J \eta + K^T \eta + C + B_1^T \lambda \quad (7)$$

where A_0, B_1, B_2, J, K and C are matrices defined by

$$\left\{ \begin{array}{l} (\forall (j, k) \in \llbracket 1, |\mathcal{E}| \rrbracket^2) A_0^{jk} = \sum_{i \in \mathcal{V}} \langle \delta_{ij}(Rv_j - Sv_j p_i), \delta_{ik}(Rv_k - Sv_k p_i) \rangle \\ (\forall (j, k) \in \llbracket 1, |\mathcal{E}| \rrbracket \times \llbracket 1, 2 \rrbracket) B_2^{jk} = \sum_{i \in \mathcal{V}} 2\delta_{ij} \langle Rv_j - Sv_j p_i, R\hat{n}_k - S\hat{n}_k p_i \rangle \\ (\forall (j, k) \in \llbracket 1, 2 \rrbracket^2) J_{jk} = \sum_{i \in \mathcal{V}} \langle R\hat{n}_j - S\hat{n}_j p_i, R\hat{n}_k - S\hat{n}_k p_i \rangle \\ (\forall j \in \llbracket 1, 2 \rrbracket) K_j = \sum_{i \in \mathcal{V}} 2 \langle R\hat{n}_j - S\hat{n}_j p_i, R\omega - S\omega p_i \rangle \\ C = \sum_{i \in \mathcal{V}} \langle R\omega - S\omega p_i, R\omega - S\omega p_i \rangle \\ (\forall j \in \llbracket 1, |\mathcal{E}| \rrbracket) B_1^j = \sum_{i \in \mathcal{V}} 2\delta_{ij} \langle Rv_j - Sv_j p_i, R\omega - S\omega p_i \rangle \end{array} \right. \quad (8)$$

Eq. 7 can be rewritten as

$$\epsilon = \begin{pmatrix} \lambda^T & \eta^T \end{pmatrix} \begin{pmatrix} A_0 & B_3 \\ B_3^T & J \end{pmatrix} \begin{pmatrix} \lambda \\ \eta \end{pmatrix} + \begin{pmatrix} B_1^T & K^T \end{pmatrix} \begin{pmatrix} \lambda \\ \eta \end{pmatrix} + C \quad (9)$$

where $B_3 = \frac{1}{2}B_2$. We have indeed $\lambda^T B_2 \eta = \lambda^T B_3 \eta + \eta^T B_3^T \lambda$.

This is also equivalent to

$$\epsilon = X^T A X + B X + C \quad (10)$$

where X, A and B are matrices give by

$$X = \begin{pmatrix} \lambda \\ \eta \end{pmatrix}, A = \begin{pmatrix} A_0 & B_3 \\ B_3^T & J \end{pmatrix}, B = \begin{pmatrix} B_1^T & K^T \end{pmatrix} \quad (11)$$

The equation (12) is the constraints defined in (2) rewriting in matrix mode using (13) and (14).

$$DX = 0 \quad (12)$$

$$D = \begin{pmatrix} D_1 & 0 \\ D_2 & 0 \\ D_3 & 0 \end{pmatrix} \quad (13)$$

$$\left\{ \begin{array}{l} (\forall (i, j) \in \llbracket 1, |\mathcal{F}| \rrbracket \times \llbracket 1, |\mathcal{E}| \rrbracket) D_1^{ij} = c_{ij} v_j^x \\ (\forall (i, j) \in \llbracket 1, |\mathcal{F}| \rrbracket \times \llbracket 1, |\mathcal{E}| \rrbracket) D_2^{ij} = c_{ij} v_j^y \\ (\forall (i, j) \in \llbracket 1, |\mathcal{F}| \rrbracket \times \llbracket 1, |\mathcal{E}| \rrbracket) D_3^{ij} = c_{ij} v_j^z \end{array} \right. \quad (14)$$

The new formulation of the problem is to find \hat{X} such that the equation (15) is respected.

$$\hat{X} = \underset{X \in \text{Ker}(D)}{\text{argmin}} (X^T A X + B X + C) \quad (15)$$

We solve Eq. (15) by Lagrange multipliers.